The Matrix

is an rectangular arrangement of elements usually ~~the~~ number e.g

$$\begin{pmatrix} 1 & 3 & 0 \\ -2 & 8 & 2 \\ 4 & 0 & -1 \\ \frac{1}{2} & 0 & 117 \end{pmatrix}$$

The above matrix is a four by 3 matrix that is it has 3 columns and 4 rows

Matrices are used in engineering to deal with several variables at once e.g

The co-ordinate of a point in a place are written as (x, y) or in space as written (x, y, z) and thus are written as column matrices

In general, an $(n \times m)$ matrix A looks like

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,m-1} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2,m-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n-1,1} & a_{n-1,2} & a_{n-1,3} & \cdots & a_{n-1,m-1} \\ a_{n,1} & a_{n,2} & a_{n,3} & \cdots & a_{n,m} \end{pmatrix}$$

a_{23} row, column

Here, the entries are denoted as ~~aij~~ a_{ii}, a_{2j} etc
In the above parties a_i no n and m are called ~~direct~~ dimension of a

i.e what is the dimension

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

Dim 2×2

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Addition

It is possible to add two matrices togeather, but only if they have the same dimensions.

$$\begin{pmatrix} 1 & 3 & 0 \\ 3 & 8 & 2 \\ 4 & 0 & -1 \\ \frac{1}{2} & 0 & 117 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 1 \\ 0 & -8 & 3 \\ 5 & 1 & 2 \\ \frac{1}{2} & 1 & -50 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 0 & -1 \\ 9 & 1 & -3 \\ 1 & 1 & 67 \end{pmatrix}$$

Note

If matrices both have the ~~sa~~ different dimension it can not be added or we say that the sum can't be defined i.e

$$\begin{pmatrix} 1 & 2 \\ 0 & -2 \\ 3 & 7 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 3 & -2 \end{pmatrix}$$

(they are not the same) so such opperation is defined to as undefined.

MULTIPLYING MATRICES

Rules:

When multipying matrices, keep ~~follo~~ the following in ~~mind~~ mind

lay first row of the matrix on top of the 2nd column of ~~2~~ the second matrix; only if they are both of the same size.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 & 3 \\ 1 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 1\cdot2+2\cdot1 & 1\cdot0+2\cdot5 & 1\cdot3+2\cdot7 \\ 3\cdot2+4\cdot1 & 3\cdot0+4\cdot5 & 3\cdot3+4\cdot7 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 10 & 17 \\ 10 & 20 & 37 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} \times \begin{pmatrix} 4 & 7 \\ -8 & 1/2 \\ 2 & 1 \end{pmatrix}$$

NOT DEFINED (NOT THE SAME (DIMENSIONS))

SIZE

Than Scalar multiplication
There is another type of multiplication of matrixes this means just multiplying each entry of the matrix by ~~a~~ number

$$A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 0 \\ 3 & 6 & 1 \end{pmatrix}$$

$= 3A$

Rules
There are some rules which matrix addition and multiplication obey
Associative: ~~(A+B~~ $(A+B)+C = A+(B+C)$
~~Com~~ Communitative: $A+B = B+A$
Distributive: $(A+B)C = AC+BC$
Associative: $(AB)C = A(BC)$

Non - Commutative : $AB \neq BA$
Moving Constant : $A(\lambda \cdot B) = \lambda(A \cdot B)$

Classwork

$$A = \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$$

AB

$$\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1\times -1 + 0\times 1 & 1\times 4 + 0\times -2 \\ 3\times -1 + 2\times 1 & 3\times 4 + 2\times -2 \end{pmatrix}$$

$$\begin{pmatrix} -1+0 & 4+0 \\ -3+2 & 12+-4 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 4 \\ -1 & 8 \end{pmatrix}$$

BA

$$\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

$-1\times 1 + 4\times 3$ $\qquad$ $-1\times 0 + 4\times 2$

$1\times 1 + -2\times 3$ $\qquad$ $1\times 0 + -2\times 2$

$-1 + 12$ $\qquad$ $0 + 8$

$-1 + -6$ $\qquad$ $0 - 4$

$$\begin{pmatrix} 11 & 8 \\ -5 & -4 \end{pmatrix}$$

Also show that $(A+B)C = AC + BC$

$$(A+B) = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$$

$$(A+B)\cdot C = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Undefined

T

The transpose of matrix A

$A = (a_{ij})_{m \times n}$, written A^t (A' or A^T)

Is the matrix of A by writing the rows of A in order as columns

$A^t = (a_{ij})_{n \times m}$

Properties of transpose

(I) $(A+B)^t = (A^t + B^t)$

(II) $(A^t)^t = A$

(III) $(KA)^t = KA^t$ for Scalar K

V A^t (A' or A^T)

(IV) $(AB)^t = B^t \cdot A^t$

Example

Using the following matrix A and B

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 5 & -4 & -3 \\ 1 & -2 & -3 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 & 6 & -2 \\ -6 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

(1) $(A+B)^t = (A^t + B^t)$

ii $(AB)^t = B^t A^t$

(1) $(A+B)^t = (A^t + B^t)$

$$A+B = \begin{pmatrix} 1 & -1 & 2 \\ 5 & -4 & 3 \\ 1 & -2 & -3 \end{pmatrix} + \begin{pmatrix} -2 & 6 & -2 \\ -1 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 5 & 0 \\ 4 & -4 & 4 \\ -1 & -1 & -3 \end{pmatrix}$$

$$(A+B)^t = \begin{pmatrix} -1 & 4 & -1 \\ 5 & -4 & -1 \\ 0 & 4 & 3 \end{pmatrix}$$

$A^t + B^t$

$$A^t = \begin{pmatrix} 1 & 5 & 1 \\ -1 & -4 & -2 \\ 2 & 3 & -3 \end{pmatrix} \qquad B^t = \begin{pmatrix} -2 & -1 & -2 \\ 6 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

$$\cancel{(A+B)}^t \begin{pmatrix} -1 & 4 & -1 \\ 5 & -4 & -1 \\ 0 & 4 & 3 \end{pmatrix}$$

$(AB)^t$

$$\begin{pmatrix} 1 & -1 & 2 \\ 5 & -4 & 3 \\ 1 & -2 & -3 \end{pmatrix} \cdot \begin{pmatrix} -2 & 6 & -2 \\ -1 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

$1 \cdot -2 + -1 \cdot -1 + 2 \cdot -2 \quad 1 \cdot 6 + -1 \cdot 0 + 2 \cdot 1 \quad 1 \cdot -2 + -1 \cdot 1$

$5 \cdot -2 + -4 \cdot -1 + 3 \cdot -2 \quad 5 \cdot 6 + -4 \cdot 0 + 3 \cdot 1 \quad 5 \cdot -2 + -4 \cdot$

$1 \cdot -2 + -2 \cdot -1 + -3 \cdot -2 \quad 1 \cdot 6 + -2 \cdot 0 + -3 \cdot 1 \quad 1 \cdot -2 + -2 \cdot$

$$\begin{pmatrix} -5 & -12 & 6 \\ 8 & 33 & 3 \\ -3 & -14 & -4 \end{pmatrix}$$

Given a number n, ~~and~~ (n by m)

~~in~~ has special properties

NOTE

If we have ($n \times n$) matrices the products will also be n by n matrix

n by n

+2.0

+3.0

NOTE: A square matrix A is said to be skew symmetric if $A = -A^t$

+-3.0

A square matrix is said to be symmetric if ~~A~~
$A = A^t$

i.e

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -4 & -2 \\ 1 & -2 & -3 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -4 & -2 \\ 1 & -2 & -3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ -3 & -5 & 10 \\ 1 & 8 & 9 \end{pmatrix}$$

$A + A^t$ is a symmetric matrix

~~At~~ $A^t = \begin{pmatrix} 1 & -3 & 1 \\ 3 & -5 & 8 \\ 5 & 10 & 9 \end{pmatrix}$

$$\overbrace{\begin{pmatrix} 1 & 3 & 5 \\ -3 & -5 & 10 \\ 1 & 8 & 9 \end{pmatrix}}^{A} + \overbrace{\begin{pmatrix} 1 & -3 & 1 \\ 3 & -5 & 8 \\ 5 & 10 & 9 \end{pmatrix}}^{A^t}$$

$$(A+A^t) = \begin{pmatrix} 2 & 0 & 6 \\ 0 & -10 & 18 \\ 6 & 18 & 18 \end{pmatrix}$$

$$(A+A^t)^t = \begin{pmatrix} 2 & 0 & 6 \\ 0 & -10 & 18 \\ 6 & 18 & 18 \end{pmatrix}$$

∴ They are Symmetrical!

~~A~~ $A - A^t$ is a Symmetrical matrix

Determinant

Let $A = (a_{ij})_{n \times n}$ be a square order

Called determinant of the matrix $|A|$

Determinant of 2X2 matrix

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$|A| = a_{11} a_{22} - a_{12} a_{21}$$

(1) Determinant of 3X3 matrix

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{23} & a_{33} \end{pmatrix}$$

← row | column

$$\cancel{|A| = a_{11}(a_{22} - a_{33}) - a_{12}(a_{21} a_{33}) a_{13}(}$$

$$|A| = a_{11}((a_{22} \times a_{33}) - a_{23} \times a_{32}) - a_{12}(a_{21} a_{33} - a_{23} a_{31}) + a_{13}(a_{21} a_{32} - a_{31} a_{22})$$

Let $A = \begin{array}{ccc} + & - & + \\ \end{array}$
$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \\ 1 & 9 & 5 \end{pmatrix}, \text{ Find } |A|$$

$|A| = \cancel{120}$

$1|6\cdot5 - 9\cdot8| - 3|2\cdot5 - 1\cdot8| + 4|2\cdot9 - 1\cdot6|$

$1|30 - 72| - 3|10 - 8| + 4|18 - 6|$

$-42 \quad -6 \quad +48$

$= 0$

PROPERTIES OF THE DETERMINANT

① The Determinant of a matrix A and its transpose (A^t) : $|A| = |A^t|$

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 4 & 3 \\ -5 & 0 & -2 \end{pmatrix}$$

$2|-8-0| - 1|0+15| - 1|0+20|$

$2|-8| - 1|15| - 1|20|$

$-16 - 15 - 20$

$|A| = -51$

$A^t = \begin{pmatrix} 2 & 0 & -5 \\ 1 & 4 & 0 \\ -1 & 3 & -2 \end{pmatrix}$

$2|-8+-0| - 0|-2+0| - 5|3+4|$

$-16 - 0 - 35$

$= -51$

$|A| = |A^t|$ are the same

Get

b Let A be a Square matrix
If A has a row (or Column) of zero then $|A| = 0$

If A has two identical rows (or Column) then $|A| = 0$

If A is a Square matrix of order n and K is a Scalar, then $|KA| = K^n |A|$

MINOR AND COFACTORS

Let $A = (a_{ij})_{n \times n}$ is a ~~Minor and Co-factors~~ Square matrix. Then M_{ij} denote a sub matrix of A with order $(n-1) \times (n-1)$ Obtained by deleting it i^{th} rows and j^{th} Column. The determinants ~~of~~ $|M_{ij}|$ is called minor of the element a_{ij} of A

The Cofactor of a_{ij} denoted by A_{ij} is equal to $\boxed{(-1)^{i+j} |M_{ij}|}$ FORMULA FOR COFACTOR

Sol

Let $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, Find the Coefficient Cofactor of A

Minor will be $M_{11}, M_{12}, M_{21}, M_{22}$.

$a_{11} = 3, \quad a_{12} = 2$

$a_{21} = 1 \quad a_{22} = 4$

$M_{11} = \begin{vmatrix} \cancel{3} & \cancel{2} \\ \cancel{1} & 4 \end{vmatrix} = 4$

$M_{12} = \begin{vmatrix} \cancel{3} & \cancel{2} \\ 1 & \cancel{4} \end{vmatrix} = 1$

~~M_{11}~~ $= M_{21} = 2, \quad M_{22} = 3$

and Cofactor will be

$$A_{ij} = (-1)^{(i+j)} |M_{ij}|$$

$$A_{11} = (-1)^{1+1} \times 4 = 4$$

$$A_{12} = (-1)^{1+2} \times 1 = -1 \times 1 = -1$$

$$A_{22} = (-1)^{2+2} \times 3 = 3$$

$$A_{21} = (-1)^{3} \times 2 = -2$$

$$A^c = \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}$$

Example 1

Let $A = \begin{pmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{pmatrix}$ Find the Cofactor of A

(i) Find the Co factor matrix

Solv

Element =

$a_{11} = 9$, $a_{12} = 2$, $a_{13} = 1$, $a_{21} = 5$, $a_{22} = -1$
$a_{23} = 6$, $a_{31} = 4$, $a_{32} = 0$, $a_{33} = -2$

$$M_{11} = \begin{vmatrix} \cancel{9} & \cancel{2} & \cancel{1} \\ \cancel{5} & -1 & 6 \\ \cancel{4} & 0 & -2 \end{vmatrix} = 2 - 0 = 2$$

M_{rc} 1st row column

$$M_{12} = \begin{vmatrix} 9 & -2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = -10 - 24 = -34$$

$$M_{13} = \begin{vmatrix} 9 & -2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = 0 + 4 = 4$$

$$M_{21} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & 4 & 6 \\ 4 & 0 & -2 \end{vmatrix} = -4 - 0 = -4$$

$M_{22} = -22$ $\quad M_{23} = -8$, $M_{31} = 13$

$M_{32} = 49$; $\quad M_{33} = -14$

Co-factor will be

$$A_{ij} = (-1)^{i+j} |M_{ij}|$$

$$A_{11} = (-1)^{1+1} \times 2 = 2$$

$A_{12} = 34$; $\quad A_{13} = 4$

$A_{21} = 4$; $\quad A_{22} = -22$; $\quad A_{23} = 8$

$A_{31} = 13$, $\quad A_{32} = -49$; $\quad A_{33} = -19$

Co-factor matrix

$$= \begin{pmatrix} 2 & 34 & 4 \\ 4 & -22 & 8 \\ 13 & -49 & -19 \end{pmatrix}$$

Adjoint Matrix

The transpose of the matrix of Co-factors of the Elements a_{ij} of A denoted $\operatorname{adj} A$ is called Adjoint of Matrix A

Example 1:

Find the Adjoint of the matrix

$$A = \begin{pmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{pmatrix}$$

$$\operatorname{Adj} A = (A_{ij})^T$$

$$A_{ij} = \begin{pmatrix} 2 & 34 & 4 \\ 4 & -22 & 8 \\ 13 & -49 & -19 \end{pmatrix}$$

$$\operatorname{adj} A = \begin{pmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{pmatrix}$$

~~Where I is the Identity matrix of the same~~ order

Theorm

~~Theory~~ Under adjoint matrix

For any square matrix $A(\text{adj } A) = (\text{adj } A)A = |A| I$

where I is an identity matrix

Proof: Let $A = (a_{ij})_{n \times n}$

Since A is a square matrix of order n, then adj A also in same order

Consider adj A also in same order

$$\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ a_{13} & a_{23} & \cdots & a_{3n} \end{pmatrix}$$

$$\text{adj } A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{pmatrix} \quad \begin{matrix} \text{— row} \\ | \text{ column} \end{matrix}$$

M = Consider the product $A(\text{adj } A)$ ~~$A(\text{Adj } A)$~~

$$A(\text{adj } A) = \begin{pmatrix} \sum_{j=1}^{n} a_{1j} A_{1j} & \sum_{j=1}^{n} a_{1j} A_{2j} & \cdots & \sum_{j=1}^{n} a_{ij} \\ \sum_{j=1}^{n} a_{2j} A_{j} & \sum_{j=1}^{n} a_{2j} A_{2j} & \cdots & \sum_{j=1}^{n} a_{2j} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=1} a_{nj} A_{ij} & \sum_{j=1}^{n} a_{nj} A_{2j} & \cdots & \sum_{j=1}^{n} a_{nj} A_{nj} \end{pmatrix}$$

as we know that

$$\sum_{j=1}^{n} a_{ij} A_{ij} = |A| \quad \therefore A(\text{Adj } A) = |A| I$$

$$\sum_{j=1}^{n} a_{ij} A_{kj} = 0 \text{ when } i \neq k$$

$$= \begin{pmatrix} |A| & 0 & \cdots & 0 \\ 0 & |A| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & |A| \end{pmatrix}$$

$$= |A| \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Consider the matrix

$$A = \begin{pmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{pmatrix}$$

2 (i) Compute $|A|$ (ii) find adj A

(iii) Verify $A(\text{adj } A) = |A| I$

$$9\begin{vmatrix} -1 & 6 \\ 0 & -2 \end{vmatrix} - 2\begin{vmatrix} 5 & 6 \\ 4 & -2 \end{vmatrix} + 1\begin{vmatrix} 5 & -1 \\ 4 & 0 \end{vmatrix}$$

$$9|0+2| - 2|-10 - 24| + 1|0+4|$$

$$9|2| - 2|-34| + 1|4|$$

$$18 + 68 + 4 = 90$$

$$\operatorname{Adj} A = (A_{ij})^T$$

$$A_{ij} = (-1)^{i+j}(M_{ij})$$

The Co-factor matrix of A was found to be

$$(A_{ij}) = \begin{pmatrix} 2 & 34 & 4 \\ 4 & -22 & 8 \\ 13 & -49 & -19 \end{pmatrix}$$

$$\operatorname{adj} A = \begin{pmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{pmatrix}$$

ii Verify $A(\operatorname{adj} A) = |A| I$

$$A(\operatorname{adj} A) = \begin{pmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{pmatrix}$$

$$\begin{pmatrix} 9\times2+2\times34+1\times4 & 9\times4+2\times-22+1\times8 & 9\times13+2\times-4 \\ 5\times2+34\times-1+6\times4 & 5\times4+-1\times-22+6\times8 & \\ 4\times2+0\times34+-2\times4 & 4\times4+0\times-22+-2\times8 & \end{pmatrix}$$

Adj. $A(\text{adj } A) = \begin{pmatrix} 90 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 90 \end{pmatrix}$

$|A|\, I = 90 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 90 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 90 \end{pmatrix}$

N find the possible value of x can take given that

$A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$

Such that $AB = BA$

$$\begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}\begin{pmatrix} 3 & 6 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}\begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$$

$$\begin{bmatrix} 3x^2 + 6 & 6x^2 + 3x \\ 3 + 6x & 6 + 3x^2 \end{bmatrix} = \begin{bmatrix} 3x^2 + 6 & 9 + 18x \\ 2x^2 + x & 6 + 3x^2 \end{bmatrix}$$

$3 + 6x = 2x^2 + x$ $\quad x = -\frac{1}{2}$

$2x^2 - 5x - 3 = 0$ $\quad$ or

$(2x^2 - 6x) + (x - 3) = 0$ $\quad x = 3$

$2(x - 3) + 1(x - 3) = 0$

Inverse of a matrix

If A and B are two matrices such that AB = BA.
Then each is said to be inverse of the order

The inverse of A is denoted by A^{-1}

Theorem I EXSISTENCE OF THE INVERSE

The necessary and sufficient Condition for a Square matrix A to have an Inverse is that determinant ~~A~~ $|A| \neq 0$

Proof:

① The necessary Condition Let A be a square matrix of order n and B is inverse of it, Then

$$AB = I$$
$$|AB| = |A||B| = I$$

Therefore $|A| \neq 0$

The sufficient Condition:

If $|A| \neq 0$, then we define the matrix B such that

$$B = \frac{1}{|A|}(\text{adj}\ A)$$

Then

$$AB = A \cdot \frac{1}{|A|}(\text{adj}\ A)$$

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Consider the matrix (Assignment)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$$

(I) Compute $|A|$ (II) find adj A

III Verify $A(\text{adj } A) = |A| I$

~~Then~~ $= \frac{1}{|A|} |A| I \cdot I$

Similarly $BA = \frac{1}{|A|} (\text{adj } A) A$

$$= \frac{1}{|A|} A (\text{Adj } A) = \frac{1}{|A|} \cdot |A| I = I$$

Thus, $AB = BA = I$

hence B is inverse of A and is given by

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

Exp 1:

Let

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix} \text{ find } A^{-1}$$

Soln

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$\text{adj } A = (A_{ij})^T$$

~~M_{11}~~ $a_{11} = 2$ $\quad a_{12} = 3$

$a_{13} = 5$ $\quad a_{21} = 4;$ $\quad a_{22} = 1$

$a_{23} = 6;$ $\quad a_{31} = 1,$ $\quad a_{32} = 4,$ $\quad a_{33} = 0$

Minor will be

$$M_{11} = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix} = 0 - 24 = -24$$

$$A_{11} = (-1)^{i+j} |M_{11}|$$

$$= (-1)^{1+1} \cdot -24 = -24$$

$$M_{12} = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix} = 0 - 6 = -6$$

$$A_{12} = (-1)^{1+2} \cdot |M_{12}| = -1 \times -6 = 6$$

$$M_{21} = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix} = -20$$

$$A_{21} = (-1)^{2+1} \cdot -20 = -20$$

$A_{22} = -5$

$A_{23} = -5$

$A_{31} = 13$

$A_{32} = 8$

$A_{33} = -10$

$\therefore$ Cofactor of matrix A

$$A_{ij} = \begin{pmatrix} -24 & 6 & 15 \\ 20 & -5 & -5 \\ 13 & 8 & -10 \end{pmatrix}$$

$$adj\, A = (A_{ij})^T = \begin{pmatrix} 24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{pmatrix}$$

$|A| = 45$

$24|-50+40| -20|-60 \quad | +13$

$$A^{-1} = \frac{1}{45}\begin{pmatrix} 24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{pmatrix}$$

SOLUTION OF SYSTEM OF LINEAR EQUATION BY MATRIX METHOD

This aspect focuses on how to find the solution of system of m linear equations in M unknowns

Consider the system of equation in unknowns $x_1, x_2 \ldots\ldots, x_n$ as $a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}$ ~~$x_n = b_1$~~

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_n = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots a_{nn}x_n = b_n$$

is Called System of linear equations ~~is~~ with n unknowns $x_1 x_2 \ldots x_n$. If Constants $b_1, b_2, \ldots b_n$ are ~~all~~ all 0 then the system is said to be a homogenous ~~Con~~

The above System Can be proved in the matrix formed

$$AX = B$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{pmatrix}$$

The matrix $A = (a_{ij})\, n \times n$

A - represent the Coefficient

$x =$ is Called

$b =$ is Called matrix of Constant

METHODS OF SOLVING SYSTEM OF LINEAR EQUATION

Consider the matrix equation

$$AX = B \quad \text{where} \quad |A| \neq 0$$

multiplying by A^{-1}

$$A^{-1}(AX) = A^{-1}B$$

$$X = A^{-1}B$$

Thus $AX = B$

~~has~~ $|A| \neq 0$

Example 2:

Solve the set of equations

$$x_1 + 2x_2 + x_3 = 4$$

$$3x_1 - 4x_2 - 2x_3 = 2$$

$$5x_1 + 3x_2 + 5x_3 = -1$$

Soln

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

$4+4+1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$1|-20+6| - 2|15+10| + 1|9+20|$$

$$1[-14] - 2|25| + 1[29]$$

$$-14 - 50 + 29$$

$$= -85$$

$$\text{adj}\,A = (A_{ij})^T$$

$$A_{ij} = (-1)^{i+j}|M_{ij}|$$

Co-factor of matrix A

$$A_{ij} = \begin{pmatrix} -14 & -25 & 29 \\ -7 & 0 & 7 \\ 0 & 5 & -10 \end{pmatrix}$$

A^{-1}

$$\text{adj}\,A = \begin{pmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{pmatrix}$$

-28

$+-7\times3+0\times4$

$$A^{-1} = \frac{1}{|A|}(\text{adj}\,A)$$

-14×2

$$= \frac{1}{-35}\begin{pmatrix} -14 & -7 & 0 \\ 25 & 0 & 5 \\ 29 & 7 & -10 \end{pmatrix}$$

$-14\times2+-7\times3$

$$X = \frac{1}{-35}\begin{pmatrix} -70 \\ -105 \\ 140 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

$x_1 = 2,\ x_2 = 3;\ x_3 = -4$

Assignment

Solve the system of linear equation using method of ~~rese~~ ~~reverse~~ inverse

$$2x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 3x_2 - x_3 = 11$$
$$2x_1 - 2x_2 + 5x_3 = 3$$

2 Using Elementary row operations (Gaussian elimination)

Suppose the square matrix is of type $m \times n$ that is

$$[A, B]$$

All the information for solving the set of information is provided by the matrix is divided by the

If we write the elements of ~~a~~ ~~A~~ B ~~and~~ within the matrix A they

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_2 & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m} & a_{n2} & & a_{nn} & b_n \end{pmatrix}$$

We then Eliminate Elements in order by ~~Substract~~ Subtracting ~~A_{21}~~ $\frac{a_{21}}{a_{11}} \times$ First row from the 2nd row

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ 0 & c_{22} & c_{23} & \cdots & c_{2n} & d_2 \\ \vdots & \vdots & \vdots & & & \vdots \\ 0 & 0 & c_{n3} & \cdots & c_{nn} & d_n \end{pmatrix}$$

Exp 3: Solve

$x_1 + 2x_2 - 3x_3$
$2x_1 - x_2 - x_3 = 11$
$3x_1 + 2x_2 + x_3 = -5$

This can be written as

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 11 \\ -5 \end{pmatrix}$$

The augmented matrix becomes

$$\begin{pmatrix} 1 & 2 & -3 & 3 \\ 2 & -1 & -1 & 11 \\ 3 & 2 & 1 & -5 \end{pmatrix}$$

Divide a_{21} by a_{11} i.e $\frac{2}{1} = 2$

Subtract 2 times the first row from Second

$\frac{2 \times 2}{1} = 4; \; -1 - 4 = -5$

$2 \times 1 = 2; \; 2 - 2$

$$\begin{pmatrix} 1 & 2 & -3 & 3 \\ 0 & -5 & 5 & 5 \\ 3 & 2 & 1 & -5 \end{pmatrix}$$

Change $a_{31} = 0$

Divide a_{31} by a_{11} i.e $\frac{3}{1} = 3$

Substract $\frac{3}{1}$ time the first row from

$\frac{3}{1} \times 1 = 3; \quad 3 - 0 = 0$

Also $= \frac{3}{1} \times 2 = 6, \quad 2 - 6 = -4$

The augumented matrix becomes

$\frac{3}{1} \times -3 = -9; \quad 1 + 9 = 10$

$\frac{3}{1} \times 3 = 9, \quad -5 - 9 - 14$

$$\begin{pmatrix} 1 & 2 & -3 & 3 \\ 0 & -5 & 5 & 5 \\ 0 & -4 & 10 & -14 \end{pmatrix}$$

~~Adding~~ Additionally Change $a_{32} = 0$

~~Now~~ Now divide a_{32} by a_{22} i.e $\frac{-4}{-5} = \frac{4}{5}$

Subtract $\frac{4}{5}$ times the second row from the third row i.e

$\frac{4}{5} \times -5 = -4; \ -4-(-4) = 0$

$\frac{4}{5} \times 5 = 4; \ 10-4 = 6$

$\frac{4}{5} \times 5 = 4, \ -14-4 = -18$

$$\begin{pmatrix} 1 & 2 & -3 & 3 \\ 0 & -5 & 5 & 5 \\ 0 & 0 & 6 & -18 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -3 & 3 \\ 0 & -5 & 5 & 5 \\ 0 & -4 & 10 & -14 \end{pmatrix}$$

Additionally,

Change $a_{32} = 0$

$AX = B$

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 18 \end{pmatrix}$$

$$-5x_2 + 5xy = 5$$
$$-5x_2 + 5(-3) = 5$$
$$x_2 = -4$$

$$x_1 + 2x_2 - 3x_3 = 3$$
$$x_1 + 2(-4) - 3(-3) = 3$$
$$x_1 - 8 + 9 = 3$$
$$x_1 = 3 - 1$$
$$x_1 = 2;\ x_2 = -4;\ x_3 = -3$$

EINGEN VALUES AND EINGEN VECTOR

In many applications of matrices ~~to~~ technological problems involving coupled oscillations and ~~comb~~ vibration, the equations of the form $A.x = \lambda.X$ occur, where $A = [a_{ij}]$ its a square matrix and λ is a number [Scalar]. ~~$x = 0$~~ Clearly $x = 0$ is a solution for any value of λ for ~~non~~ non-trivial solution that is ~~$x =$~~ $X \neq 0$, the values of λ are called the ~~Eig~~ Eigen Value, Characteristics value or latent roots and the corresponding solutions of the given equations $AX = \lambda X$ are called the Eingen Vectors or Characteristics vectors of A

Consider
$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_3 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = \lambda x_1 \quad -$$
$$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = \lambda x_2 \quad -$$
$$\vdots \quad \vdots \quad \vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots a_{nn}x_n = \lambda x_n$$

$$A \cdot X = \lambda X$$
$$\cancel{AX =}$$
$$AX - \lambda X = 0$$
$$\cancel{A \; X(A-\lambda) = 0}$$
$$(A - \lambda I)X = 0$$

Find the Eigen Values of the matrix

$$A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$

Soln

$A \cdot X = \lambda X$ i.e. $(A - \lambda I)X = 0$

Characteristic Determinant: $|A - \lambda I|$

$$= \begin{vmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$\cancel{4 \quad + 2(-1)}$$
$$(4-\lambda)(1-\lambda) - 2(-1) = 0$$
$$4 - 4\lambda - \lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 - 5\lambda + 6 = 0$$
$$(\lambda-2)(\lambda-3) = 0$$
$$\underbrace{\lambda = 2 \text{ or } \lambda = 3}_{\text{Eigen Values}}$$

$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

Example 2

Find the eigen values of the matrix

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 4 & -2 \\ 2 & 10 & -5 \end{pmatrix}$$

The Characteristics determinant

$$\begin{vmatrix} 2-\lambda & 3 & -2 \\ 1 & (4-\lambda) & -2 \\ 2 & 10 & (-5-\lambda) \end{vmatrix}$$

$$2-\lambda \,|(4-\cancel{\lambda})\cancel{-}(-5-\lambda)| -3 |1(-5-\lambda)-(-4)|$$
$$-2\,|10-2(4-\lambda)|$$

$$\cancel{2-\lambda| -20 - 4\lambda}$$
$$2-\lambda| \cancel{-20}\,(4-\lambda)\cancel{(}5+\lambda) \; -3\,|-5-\lambda+4|$$
$$-2|10-8+\lambda|$$
$$2-\lambda\,|9| \; -3\,|-1-\lambda\,| -2|\,2+\lambda|$$
$$18-9\lambda + 3 + 3\lambda - 4 \cancel{-4} - 2\lambda$$

$(2-\lambda)\lambda(\lambda+1) - (1+\lambda)$

$= (1+\lambda)(2\lambda - \lambda^2 - 1)$

$= -(1+\lambda)(1-\lambda)^2$

Characteristics equation

$(1+\lambda)(1-\lambda^2) = 0$

$\lambda = -1;\ \lambda = 1;\ \lambda = 1$

Each Eigen Values Obtained has Corresponding ~~to the~~ ~~a~~ Solution of x Called an ~~Egine~~ Eigen Vector in matrices the term vector indicate a row matrix or Column matrix

Consider the equation

$AX = \lambda X$ where

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4-\lambda & 1 \\ 3 & 2-\lambda \end{pmatrix}$$

$(4-\lambda)(2-\lambda) - 3(1)$

$8 - 4\lambda - 2\lambda + \lambda^2 - 3$

$8 - 6\lambda + \lambda^2 - 3$

$5 - 6\lambda + \lambda^2$

product

5

λ

$-5\ -1$

$\cancel{5 - 5\lambda - \lambda + \lambda^2}$

$\lambda^2 - 6\lambda - 5$ $\quad \cancel{5(1-\lambda) - \lambda(1-\lambda)}$

$\lambda^2 - \lambda - 5\lambda - 5$ $\quad \cancel{(5-\lambda)(1-\lambda)}$

$\lambda(\lambda-1) - 5(\lambda-1)$

$(\lambda-5)(\lambda-1)$ $\quad \lambda = 5$ and 1

for $\lambda_1 = 1$, the equation $AX = \lambda X$ becomes

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 1 \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$4x_1 + x_2 = x_1$

$3x_1 + 2x_2 = x_2$

$x_2 = x_1$

$x_2 = 2$

Either of these gives you $x_2 = -3x_1$

$4x_3 = 5x_2$

This result in the case that whichever value

$\therefore$ The Eigen Vector is $x_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is a solution

for $\lambda = 1$

$\begin{pmatrix} x_3 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5}{4} \end{pmatrix}$

for $\lambda = 5$

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 5 \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$4x_1 + x_2 = 5x_1$

$3x_1 + 2x_2 = 5x_2$

$x_1 = x_2$

$x_1 = x_2$

$x_1 = 1;\ x_2 = 2$

$X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is a solution for $\lambda = 5$

Ans

Determine the eigen values and ~~eg~~ eigen vectors for the matrix A

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}$$

$x_1 = 3$
$x_2 = 2$