COURSE OUTLINE

ENG 281 - Engineering Mathematics I

1 Limits and Continuity

2 Differentiation

Techniques of differentiation

~~[illegible]~~ Applications of differentiation

Equation of a straight line

Tangents and Normals to a Curve at a given

Curvatures

3 Integration

Techniques of integration

Applications of integration

Areas under Curves

Volume of Solids of Revolution

4 Linear Algebra

Matrices

Matrix notations

Equal matrices

Addition and Subtraction of

Matrices

Transpose of a matrix

Special matrices

Determinant of a Square matrix

Inverse of a Square matrix Linear System

~~Sys~~ Systems

Solution of a set of linear equations

Eingen Values and eigenvectors

5 Introduction to MATLAB and MATHCAD programming

General Introduction

Matrices

Entering a matrix into MATLAB and MATHCAD

Matrix notation definition

Addition and Subtraction of matrices

Scalar Multiplication of two matrices

Elemental multiplication of two matrices

Transpose of a matrix

Determinant of a matrix

Inverse of a matrix

Solution of a set of linear equations

Differentiating a functions

Differentiating a functions with respect to x

Differentiating a function Containing ~~the~~ Pi, exponential and trigonometry functions.

Differentiating a functions with respect to different variables

Integrating a function: Indefinite integral, ~~different variables~~ definite integral

Integrating a function

Plotting graph: Putting x and y labels, legends for multiline plot, inserting tick marks

Finding the ~~region~~ eigen values of a matrix

SUGGESTED TEXTBOOKS

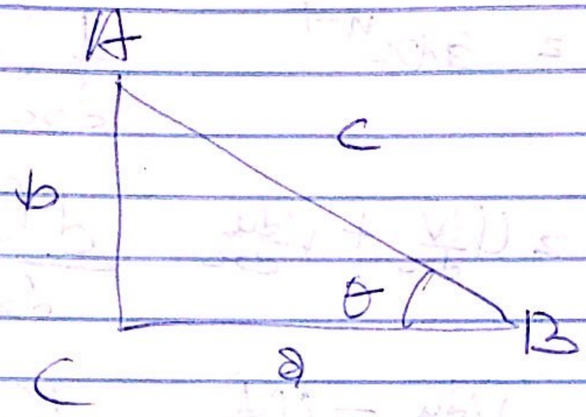
1 ENGINEERING MATHEMATICS BY K.A. STROUD

2 CALCULUS WITH ANALYTIC GEOMETRY BY E.W. SWOKOWSKI.

JUST IN CASE

TRIGONOMETRY

NOTE



$\sin\theta = \frac{Opp}{hyp} = \frac{b}{c}$

$\cos\theta = \frac{Adj}{hyp} = \frac{a}{c}$

$\tan\theta = \frac{Opp}{Adj} = \frac{b}{a}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} = \frac{c}{b}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{Adj} = \frac{c}{a}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{Adj}{Opp} = \frac{a}{b}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\sin A+B = \sin A\cos B + \cos A\sin B$

$\sin A-B = \sin A\cos B - \cos A\sin B$

$\cos A+B = \cos A\cos B - \sin A\sin B$

$\cos A-B = \cos A\cos B + \sin A\sin B$

$\tan A+B = \frac{\tan A + \tan B}{1 - (\tan A\tan B)}$

$\tan A-B = \frac{\tan A - \tan B}{1 + (\tan A\tan B)}$

$\cos 2A = \cos(A+A)$

$= \cos^2 A - \sin^2 A$

$\sin(A+A) = 2\sin A\cos A$

$$\frac{dy}{dx}(ax^n) = anx^{n-1}$$

$$\frac{dy}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\left.\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ \frac{dy}{dx} &= \frac{dy}{dt} \div \frac{dx}{dt} \end{aligned}\right\} \text{FUNCTION OF A FUNCTION}$$

$$\frac{dy}{dx} = 1 \div \frac{dx}{dy}$$

$$\frac{dy}{dx}(\sin\theta) = \cos\theta$$

$$\frac{dy}{dx}(\cos\theta) = -\sin\theta$$

$$\frac{dy}{dx}(\tan\theta) = \sec^2\theta$$

$$\frac{dy}{dx}(\arcsin\theta) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}(\arccos\theta) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x}$$

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(e^{ax})}{dx} = ae^{ax}$$

$$\frac{\cancel{d(\theta\sin x)}\ d(x\sin\theta)}{dx} = \cancel{x\theta}\cos\theta$$

$$\lim_{x\to\infty}\left\{\frac{1}{x}\right\} = 0$$

and

$$\lim_{x\to\infty}\left\{\frac{1}{x^2}\right\} = 0$$

LIMIT AND CONTINUITY
LIMITS

Some more definition

Having a particular starting point to an ending point.

A limit is the value of function or sequence approaches as the input (or index) approaches some value

$\lim_{x \to x_0} f(x) = L$ exists if the following conditions are satisfied

1 $f(x)$ is defined in an open interval containing x_0 but not necessarily at x_0.

2 $\lim_{x \to x_0^+} f(x)$ (RHS) and $\lim_{x \to x_0^-} f(x)$ exists, and

3 $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = L$

Example I

determine the limit of the following

a $\lim_{x \to 1} \frac{x^2 - 1}{x + 1}$

b $\lim_{x \to 3} (x^3 - 2x + 6)$

c $\lim_{x \to 2} \left(\frac{x^3 - 3x + 6}{-x^2 + 15}\right)$

Soln

a $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

$f(x) = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\lim_{x \to 1} (x+1)$

$\lim_{x \to 1} (1+1)$

$\lim_{x \to 1} 2 = \underline{2}$

b $\lim_{x \to 3} (3^2 - 2(3) + 6) = 27$

c $\lim_{x \to 2} \left(\frac{2^3 - 3 \times 2 + 6}{-(2)^2 + 15} \right)$

$\frac{8 - 6 + 6}{-4 + 15} = \frac{8}{11}$

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4 Obtain the limit

a $\lim_{x \to \infty} \dfrac{x^3+3x+6}{x^5+2x^2+9}$

b $\lim_{x \to \infty} \dfrac{2x^2-2x+3}{x^2+4x+4}$

Soln

a $\lim_{x \to \infty} \dfrac{x^3+3x+6}{x^5+2x^2+9}$

$$\dfrac{\dfrac{x^3}{x^5}+\dfrac{3x}{x^5}+\dfrac{6}{x^5}}{\dfrac{x^5}{x^5}+\dfrac{2x^2}{x^5}+\dfrac{9}{x^5}}$$

$$\lim_{x \to \infty} \dfrac{\dfrac{1}{x^2}+\dfrac{3}{x^4}+\dfrac{6}{x^5}}{1+\dfrac{2}{x^3}+\dfrac{9}{x^5}}$$

$$= \dfrac{\dfrac{1}{\infty^2}+\dfrac{3}{\infty}+\dfrac{6}{\infty^5}}{1+\dfrac{2}{\infty^2}+\dfrac{9}{\infty^5}}$$

b $\lim_{x \to \infty} \dfrac{\dfrac{2x^2}{x^2}-\dfrac{2x}{x^2}+\dfrac{3}{x^2}}{\dfrac{x^2}{x^2}+\dfrac{4x}{x^2}+\dfrac{9}{x^2}}$

$\dfrac{1}{x}=0$

$$\dfrac{2-\dfrac{2}{x}+\dfrac{3}{x^2}}{1+\dfrac{4}{x}+\dfrac{9}{x^2}} = \dfrac{2}{1} = 2$$

FC

APPLICATION OF L'HOPITAL'S RULE

We can apply L'Hopital's Rule, whenever direct substit-ion of a limit hits an indeterminate form

L'HOPITAL'S RULE STATES THAT

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$,

then $\lim_{x \to a} \frac{f'(x)}{g'(x)}$

This means that limit of a (ocicent of a function (That is, an algebraic fraction) is equal to the ~~that~~ limit of their derivatives.

It is important to notes that L'hopital's Rule fits $f(x)$ ~~and~~ g and $g(x)$ as independent function and it is not in the application of ~~quotient~~ rule quotient rule

~~So~~ Note

Numerator and denominator are usually differentted ~~before the limit~~ is taken separately before the limit is taken

Sometimes L'hopital rule may be applied more than once

$$\frac{x^2 - 4}{x - 2} \text{ as } x \to 2$$

Soln

$$\cancel{x^2} \quad \frac{(2)^2 - 4}{2 - 2} = \frac{4 - 4}{2 - 2} = \frac{0}{0}$$

Furthermore

$f(x) = x^2 - 4$

$f'(x) = 2x$

$g(x) = x - 2$

$g'(x) = 1$

$\therefore \lim_{x \to 2} \frac{f'(x)}{g'(x)} = \frac{2x}{1} = 2 \times 2 = 4$

Find

$\lim_{x \to -2} \frac{x-2}{x^2+3x+2}$ by L'Hospital's rule

$\frac{2-2}{4+6+2} = \frac{0}{12}$

$\frac{1}{2x+3}$ $\frac{2}{4+3} = \frac{2}{7}$

$\frac{-2}{2(-2)+3} = \frac{-2}{-4+3} = \frac{-2}{-1}$

$= 1$

Find

$\lim_{x \to \infty} \frac{2x^2 - 5x + 1}{3 + x + 6x^2}$

$$\frac{\frac{2x^2}{x^2} - \frac{5x}{x^2} + \frac{1}{x^2}}{\frac{3}{x^2} + \frac{x}{x^2} + \frac{6x^2}{x^2}} = \frac{1}{3}$$

~~D39~~ D31

Assignment

Find the following limit

1 $\lim_{x \to -7} (2x+5)$

2 $\lim_{x \to \infty} \dfrac{x^5+3}{x^2+4}$

3 $\lim_{t \to 1} \dfrac{t^2+t-2}{t^2-1}$

4 $\lim_{u \to \infty} \dfrac{(2u+1)^4}{(3u^2+1)^2}$

LEFT AND RIGHT - HAND LIMITS

In some cases you let x approach the number a from the left or the right rather than both sides at once rather than usual

a Right hand limit:

$f(x)$ is defined on an interval (c, b) for $c < b$ we say that $\lim_{x \to c^+}$ means

for every number $\epsilon > 0$, there is a number, such that if $c < x < c$ + number

Left hand limit

Suppose $f(x)$ is defined on an interval (a, c) for $c < b$

$\lim_{x \to c^-} f(x) = L$ means that

for every number $\epsilon > 0$, ~~no~~ number, such that if c − number $< x < c$

~~then~~ $\epsilon > [f(x)] - L$

~~In the~~ Suppose $f(x)$ is defined on a - Open interval Containing C

$\lim_{x \to c} f(x)$ is defined if and only if

$\lim_{x \to c^+}$ and $\lim_{x \to c^-}$

are defined and equal

Example $\begin{cases} x^3 - 4 & \text{for } x < 2 \quad \text{L.H.S} \\ 2x & \text{for } x \geq 2 \quad \text{R.H.S} \end{cases}$

a . What is $\lim_{x \to 2^-} f(x)$? LHS

b What is $\lim_{x \to 2^+} f(x)$? RHS

Solu

$\lim_{x \to 2^-} x^3 - 4 = 2^3 - 4 \Rightarrow 8 - 4 = 4$

$\lim_{x \to 2^+} 2x = 2 \times 2 = 4$ $\quad -4 < x < 4$

$4 = 4$

Does this function exist?

$f(x) = \begin{cases} x^5 - 12 & \text{for } x < 2 \\ (x+1)^3 - 8 & \text{for } x \geq 2 \end{cases}$

$\text{L.H.S} = \lim_{x \to 2^-} x^5 - 12$

$2^5 - 12 \Rightarrow 32 - 12$

~~4 4~~ $= 20$

$$\lim_{x \to 2^+} (x+1)^3 - 8 \Rightarrow (2+1)^3 - 8$$

$$(3)^3 - 8 =$$

$$27 - 8 = 19$$

~~Exp~~

they are not equal
so it doesn't exist

Example

A function is defined by

$$f(x) = \begin{cases} kx + 7 & \text{if } x \geq 2 \\ x^2 + 19 & \text{if } x < 2 \end{cases}$$

$$\lim_{x \to 2^+} kx + 7$$

$$2k + 7$$

$$\lim_{x \to 2^-} x^2 + 19 \Rightarrow 2^2 + 19$$

$$4 + 19 = 23$$

$$23 = 2k + 7$$

$$23 - 7 = 2k$$

$$16 = 2k$$

$$k = 8$$

LIMITS INVOLVING TRIGONOMETRIC FUNCTIONS

The Trigonometric functions Sine and Cosine have four important limit properties

(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(2) $\lim_{x \to c} \sin x = \sin c$

(3) $\lim_{x \to c} \cos x = \cos c$

4 $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

<u>Note</u> $\cos 0 = 1$ $\sin 0 = 0$

These properties can be used to evaluate many limit problems involving basic trigonometric function

Example:

Evaluate $\lim_{x \to 0} \frac{\cos x}{\sin x - 3} \Rightarrow \frac{\cos 0}{\sin 0 - 3} = \frac{1}{-3}$

Assignment

Evaluate

① $\lim_{x \to 0} \cot x$

② $\lim_{x \to 0} \frac{\sin 4x}{x}$

LIMITS OF FUNCTION OF TWO OR MORE VARIABLES

Computing limits using the analytical method using limit rules and theorem. The rules and theorems are similar to those used with functions of one variable

Let us assume that L, M and K are real numbers and that

$\lim f(x,y) = L$ and $\lim_{(x,y) \to (a,b)} g(x,y) = M$, then

the following hold

① $\lim_{(x,y) \to (a,b)} x = a$; $\lim_{(x,y) \to (a,b)} y = b$

if C is any Constant

$\lim_{(x,y) \to (a,b)} C = C$

2 Sum and difference rules:

$$\lim_{(x,y)\to(a,b)} [f(x,y) \pm g(x,y)] = L \pm M$$

3 Constant multiple rule:

$$\lim_{(x,y)\to(a,b)} [k f(x,y) = kL$$

4 Product rule

$$\lim_{(x,y)\to(a,b)} [f(x,y) \cdot g(x,y)] = LM$$

5 Quotient rule:

$$\lim_{(x,y)\to(a,b)} \left[\frac{f(x,y)}{g(x,y)}\right] = \frac{L}{M}$$

6 Power rule:

If R and S are integers with no common factors

$$\lim_{(x,y)\to(a,b)} [f(x,y)]^{r/s} = L^{r/s}$$

~~To Find~~

Example 1

Find $\lim_{(x,y)\to(1,2)} x^6 y + 2xy$

$= 1^6 2 + 2(1)(2) = 6$

Ex 2: Find $\lim_{(x,y)\to(1,1)} \frac{x^2y}{x^4+y^2}$

$$\frac{1^2\times 1}{1^4+1^2} = \frac{1}{2}$$

Exp 3: Find $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x-y}$

$$\frac{(x-y)(x^2+xy+y^2)}{(x-y)}$$

$$\cancel{\frac{0^3-0^3}{0-0}} = \cancel{0}$$

$$x^2+xy+y^2$$

$$\frac{3x^2}{1} \quad \frac{3y^2}{1}$$

$$0^2+\cancel{0}0+0^2 = 0$$

Ex 4: Find $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$

$$\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$$

$$\frac{x(x-y)\times\sqrt{x}+\sqrt{y}}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}$$

$$\frac{x(x-y)\times\sqrt{x}+\sqrt{y}}{x+\sqrt{xy}+\sqrt{xy}+y}$$

$$\frac{(x^2+xy)(\sqrt{x}+\sqrt{y})}{x+2\sqrt{xy}+y}$$

~~Limi~~ Limit Along a Path

One way to prove that

$$\lim_{(x,y)\to(a,b)} f(x,y)$$

is to prove that this limit has different values along two different paths

When computing the limit along the paths use the relation which defines the paths e.g when computing the limit along the path $y=x$ replace y by 0 in the function.

If computing the limit along the path $y=x$ replace y by x in the function

Consider the function
$$f(x,y) = \frac{y}{x+y-1}$$

The goal is to try to find

$$\lim_{(x,y)\to(0,0)} \frac{y}{x+y-1}$$

Soln

Use the notation $\left.\frac{y}{x+y-1}\right|_{y=0}$

$$\left.\frac{y}{x+y-1}\right|_{y=0} = \frac{0}{x-1} = 0$$

Also, note that along the path $y=0$

y is constant $(x,y) \to (1,0)$ can be replaced

~~x~~ app $(x, y) \longrightarrow 1$ ~~the~~ $\therefore$ ~~limit~~

$$\therefore \lim_{(x,y)\to(1,0)} \frac{y}{x+y-1} = \lim_{x\to 1} = 0$$

lim along the ~~lo~~ path $x=1$

limit along the path $x=1$, we have

$$\left.\frac{y}{x+y-1}\right|_{x=1} \quad \frac{y}{1+y-1}$$

$$= \frac{y}{y} = 1$$

$$\lim_{(x,y)\to(1,0)} \frac{y}{x+y-1} = \lim_{y\to 0} 1 = 1$$

The limits are different therefore the limit of this function does not exsist.

Assignment

1. Prove that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exsist

2. Does $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y^2}$ exist?

CONTINUITY

The function $f(x)$ is said to be continous at a point ~~(~~ $(c, f(c))$ if each of this conditions is satisfied:

1. $f(c)$ exsists ~~(~~ (c is in the domain of f)

2. $\lim_{x\to c} f(x)$ exsists,

3. $\lim_{x\to c} f(x) = f(c)$

Geometrically, this means that there is no gap, split or mixing point for $f(x)$ at c and ~~that~~ the pencil could be moved along the graph $f(x)$ through $c, f(c)$ without lifting it of

The function is said to be continuous $(c, f(c))$ from the right ~~at~~ if $\lim_{x \to c} f(x) = f(c)$

and continuous at $(c, f(c))$ from the left if $\lim_{x \to c} f(x) = f(c)$

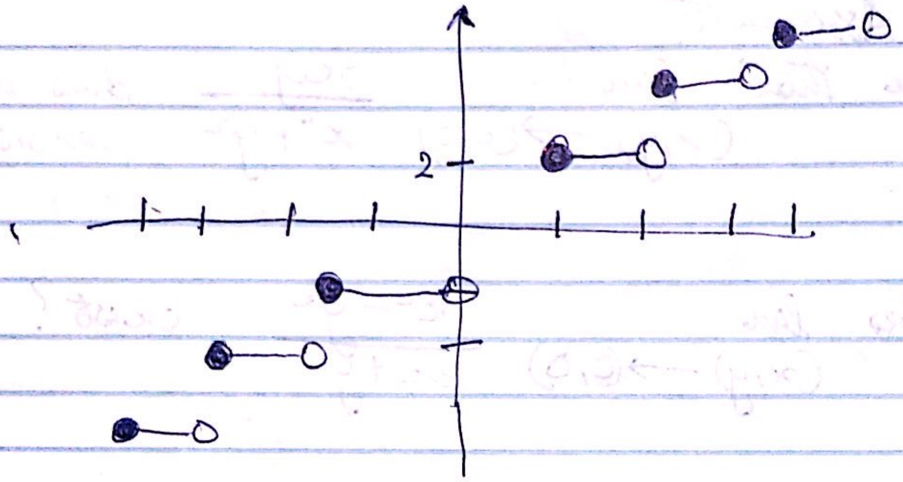

Fig 1: The graph of the greatest integer function $y = \lfloor$

~~Special~~
Example 1

discuss the continuity of $f(x) = 2x + 3$ at $x = -4$

Soln

(1). ~~$f(x)$~~ $f(-4) = 2(-4) + 3 = -5$ $\quad f(c)$

(2) $\lim_{x \to -4} f(x) = \lim (2x+3) = -5$ $\quad$ NOTE $\lim_{x \to c} f(x)$

(3) $\lim_{x \to -4} f(x) = f(-4)$

Hence, f is continuous at $x = 4$

Ex 2
Discuss the Continuity of $f(x) = \begin{cases} \frac{x^2-4}{x-2} & x \neq 2 \\ 4, & x = 2 \end{cases}$

when the definition of Continuity is applied to $f(x)$ at $x=2$

Soln

(1) $f(2) = 4$ $\qquad \frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$x+2$

2 $\lim_{x \to 2} f(x) = \lim_{x\to 2} \frac{x^2-4}{x-2} = 4$

CLASSWORK

Discuss the Continuity of $f(x)$

$\begin{cases} 5-2x, & x < -3 \text{ LHS} \\ & \text{at } x = -3 \\ x^2+2, & x \geq -3 \text{ RHS} \end{cases}$

(1) $f(x)$ $f(-3) = 5-2(-3)$ LHS
$5+6 = 11$

$f(-3) = (-3)^2+2$
$9+2 = 11$

(2) $\lim_{x\to -3^-} f(x) = \lim 5-2x = 11$

$\lim_{x\to -3^+} f(x) = \lim x^2+2 = 11$

$f(x) = f(-3) = 11$